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KINETICS OF THE NEUTRALIZATION OF STATIC
ELECTRICITY IN APPARATUS CONTAINING TWO-
PHASE SYSTEMS OF GAS AND SOLID PARTICLES

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The mechanisms of neutralization of static electricity of disperse systems when ions of the opposite sign are present in the gaseous medium are discussed.

In the design of neutralizers of static electricity for industrial apparatus containing two-phase systems of gas and solid particles the calculation of the performance of these neutralizers acquires great importance. The value of this parameter can be determined by analyzing the process of neutralization of the charges of particles of the product being treated in the presence of ions of the opposite sign in the gas-air medium of the working volume of the apparatus. Let us consider a charged spherical particle over which a stream of ions of the opposite sign flows (Fig. 1).

Under these conditions the variation of the charge of a particle is described by the equation

$$dq/dt = e \int_S \bar{j}_n d\bar{S}. \quad (1)$$

The flux density is defined as [1]

$$j_n = -nkE_{\Sigma} + D \text{grad } n. \quad (2)$$

The neutralization of the charges of a particle takes place in accordance with Eq. (2) as a single process, but to simplify the solution we will consider two mechanisms separately: a) ion motion directed toward the surface of the particle due to the electric field; b) ion motion due to diffusion.

Let us consider the first mechanism. The following forces act on an ion which is near a charged particle:

$$\bar{E}_{\Sigma} = \bar{E}_e + \bar{E}_{\text{pol}} + \bar{E}_c + \bar{E}_{\text{m}^+} + \bar{E}_{\text{equ}}. \quad (3)$$

Let us find the components of these voltages on the vector $d\bar{S}$:

$$1) E_e = |\bar{E}_e| \cdot \cos \theta = \frac{qNr_i}{2\epsilon_0} \cos \theta. \quad (4)$$

To simplify the calculations we assume that \bar{E}_e is the same at different points

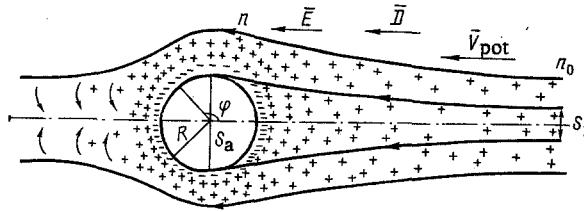


Fig. 1. Flow of ionized air over a particle.

$$2) E_{\text{pol}} = 2E_m \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \cos \theta, \quad (5)$$

$$3) E_C = q/4\pi\epsilon_0 R^2, \text{ where } R = r_p (1 + \nu). \quad (6)$$

4) E_m at a distance l from the center of the particle:

$$E_m = \frac{l}{4\pi\epsilon_0 r_p^2} \cdot \frac{1}{\gamma^3 (\gamma^2 - 1)^2}, \text{ where } \gamma = l/r_p. \quad (7)$$

The role of the mirror-image force consists in the fact that the particle is struck not only by ions whose trajectory of motion intersects the particle's surface but also those whose trajectories lie at a certain distance from it. Thus, the role of this force comes down to a kind of artificial increase in the particle diameter.

As calculations show, this increase reaches 30% for a particle $0.1 \mu\text{m}$ in diameter with $E_e \approx 3.0 \cdot 10^5 \text{ V/m}$, while for a $1\text{-}\mu\text{m}$ particle it is already negligible. Since in practice the values of E_e and d_r are far larger than the values indicated above, the quantity E_m can be neglected.

$$5) E_{\text{equ}} = V_{\text{pot}}/k. \quad (8)$$

Thus, for the projection of the total field strength onto \overline{dS} we obtain

$$E_{\Sigma} = -\frac{qNr_i}{2\epsilon_0} \left(1 + 2 \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \right) \cos \theta + q/4\pi\epsilon_0 R^2 + V_{\text{pot}}/k. \quad (9)$$

If we use Pauthenier's method [2] for an isolated spherical particle, then φ_a is the angle outside which E_{Σ} is equal to zero. Then

$$\cos \varphi_a = -q/4\pi\epsilon_0 R^2 \left[-\frac{qNr_0}{2\epsilon_0} \left(1 + 2 \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \right) \cos \theta \right] + V_{\text{pot}}/k. \quad (10)$$

Considering that the space charge is $\rho = ne$, and taking $\text{grad } \rho_{\text{pol}} = 0$, we obtain

$$\partial^2 q / \partial S \partial t = -k E_{\Sigma} \rho_{\text{pol}}. \quad (11)$$

Let us solve this equation for $E_e \approx 0$, for the case when the particle is at the center of a cylindrical apparatus with a uniformly distributed charge, for example. Then

$$E_{\Sigma} = \frac{V_{\text{pot}}}{k} \cos \varphi + q/4\pi\epsilon_0 R^2, \quad (12)$$

$$\cos \varphi_a = \frac{q}{4\pi\epsilon_0 R^2} \frac{k}{V_{\text{pot}}}.$$

The equation takes the form

$$\partial^2 q / \partial S \partial t = -\left(\frac{V_{\text{pot}}}{k} \cos \varphi + q/4\pi\epsilon_0 R^2 \right) k n e. \quad (13)$$

Since $dS = 2\pi r_p^2 \sin \varphi d\varphi$, Eq. (13) takes the form

$$\partial^2 q / \partial \varphi \partial t = -2\pi r_p^2 \sin \varphi k n e \left(\frac{V_{\text{pot}}}{k} \cos \varphi + q/4\pi\epsilon_0 R^2 \right). \quad (14)$$

In solving Eq. (11) one must allow for the following condition: it follows from Eq. (4) that the angle φ varies within different ranges as a function of the relation between E_C and E_{equ} ; for example, $0 \leq \varphi \leq \pi$ when

$$q_{br} \geq q \geq q_{crit} = BV_{pot}/k,$$

where q_{br} is the maximum possible value of the charge on the surface of a spherical particle in accordance with the electric strength $B = 4\pi\epsilon_0 R^2$ of the surrounding medium, and BV_{pot}/k is the conditional charge, the field of which imparts a velocity $V = V_{pot}$ to the ion, i.e., owing to the strong electric field the entire surface is covered with ions. Under these conditions integration of the equation for φ in the limits from 0 to π gives

$$dq/dt = -2Aq/B, \quad (15)$$

where $A = 2\pi r_p^2 k n e$, and further, integrating over t and considering that the charge is $q = q_{init}$ at $t = 0$, we obtain

$$q(t) = q_{init} \exp\left(-\frac{2A}{B} t\right). \quad (16)$$

Equation (7) allows us to calculate the time t_{crit} in which the charge q_{init} decreases to the value q_{crit} :

$$t_{crit} = -\frac{\epsilon_0 R^2}{k n e r_p^2} \ln \frac{k q_{init}}{4\pi\epsilon_0 R^2 V_{pot}}. \quad (17)$$

As the charge of a particle varies from q_{crit} to zero the angle φ varies within the limits of $0 \leq \varphi \leq \arccos(-q/q_{crit})$. Integrating Eq. (15) over t , we obtain

$$\partial q/\partial t = -\frac{A}{2Bq_{crit}} (q + q_{crit})^2. \quad (18)$$

We solve Eq. (18) with the initial conditions

$$t = 0, \quad q = q_{crit},$$

$$q(t) = 2q_{crit} / (1 + \frac{A}{B} t) - q_{crit}. \quad (19)$$

In accordance with Eq. (19), the charge of a particle is neutralized when $q_{init} \geq q_{crit}$. If the initial charge of a particle is $q_{init} < q_{crit}$, the neutralization takes place in accordance with the equation

$$q(t) = q_{crit} \left(\frac{1}{\frac{A}{2B} t + \frac{1}{1 + q_{init}/q_{crit}}} - 1 \right). \quad (20)$$

From Eq. (10) we obtain the total neutralization time:

$$t_{neutr} = \frac{2B}{A} \frac{1}{1 + q_{init}/q_{crit}}. \quad (21)$$

Let us consider the solution of Eq. (13) in the presence of an external electric field, i.e., let us consider the case when a particle does not lie at the center of the apparatus. When the apparatus has a cylindrical shape [$l_{app}(h) \gg d_{app}$], the external field strength is

$$E_e = q N r_{app} / 2\epsilon_0. \quad (22)$$

When $r = R_{app}$ (the particle is at the wall of the apparatus), $E_e = E_e \max$. Then Eq. (5) takes the form

$$\partial^2 q / \partial \varphi \partial t = -2\pi r_p^2 k n e \sin \varphi \left\{ \left[-\frac{q N R_{app}}{2\epsilon_0} \left(1 + 2 \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \right) \cos \theta + V_{pot} / k \right] \cos \varphi + q / 2\epsilon_0 \pi R^2 \right\}. \quad (23)$$

Here, just as in the preceding case, when $q \geq q_{crit}$ the angle is $0 \leq \varphi \leq \pi$, while when $q_{crit} \geq q \geq 0$ it is

$$0 \leq \varphi \leq + \arccos \frac{-q}{4\pi\epsilon_0 R^2 \left[-\frac{q N R_{app}}{2\epsilon_0} \left(1 + 2 \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \right) \cos \theta + V_{pot} / k \right]}.$$

In this case the value of q_{crit} is

$$q_{crit} = 4\pi\epsilon_0 R^2 V_{pot} / k (-1 + 4\pi\epsilon_0 R^2 C), \quad (24)$$

where

$$C = \frac{N R_{app}}{2\epsilon_0} \left(1 + 2 \frac{\epsilon - \epsilon_1}{\epsilon + 2\epsilon_1} \right) \cos \theta.$$

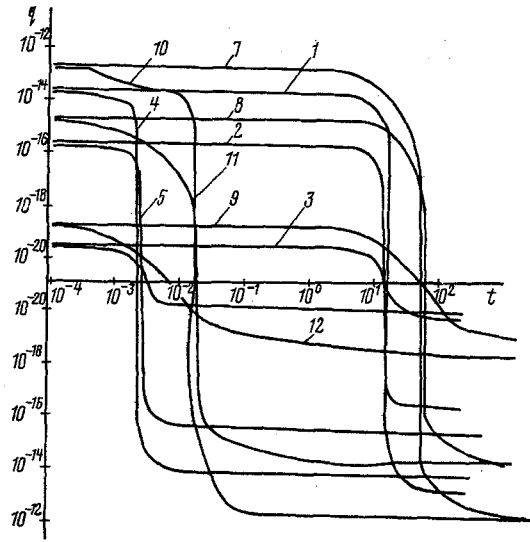


Fig. 2. Variation of the charge q , C, of a particle with time t , sec (first neutralization mechanism): 1) $d = 0.5 \cdot 10^{-3}$ m, $n_e = 0.99 \cdot 10^{-8}$ m $^{-3}$, $V_{pot} = 1$ m/sec; 2) $0.5 \cdot 10^{-4}$, $0.99 \cdot 10^{-8}$, and 1, respectively; 3) $0.5 \cdot 10^{-6}$, $0.99 \cdot 10^{-8}$, and 1; 4) $0.5 \cdot 10^{-3}$, $0.99 \cdot 10^{-4}$, and 1; 5) $0.5 \cdot 10^{-4}$, $0.99 \cdot 10^{-4}$, and 1; 6) $0.5 \cdot 10^{-6}$, $0.99 \cdot 10^{-4}$, and 1; 7) $0.5 \cdot 10^{-3}$, $0.99 \cdot 10^{-8}$, and 5; 8) $0.5 \cdot 10^{-4}$, $0.99 \cdot 10^{-8}$, and 5; 9) $0.5 \cdot 10^{-6}$, $0.99 \cdot 10^{-8}$, and 5; 10) $0.5 \cdot 10^{-3}$, $0.99 \cdot 10^{-4}$, and 5; 11) $0.5 \cdot 10^{-4}$, $0.99 \cdot 10^{-4}$, and 5; 12) $0.5 \cdot 10^{-6}$, $0.99 \cdot 10^{-4}$, and 5.

By solving Eq. (5) with allowance for the value of E_e , we obtain an expression for the time during which the charge of a particle decreases from $q_{init} \geq q_{crit}$ to zero:

$$t_{tot} = \frac{B}{2A} \ln \frac{q_{init}}{q_{crit}} + \frac{2B^2}{A(1+BC)^2} \left\{ C \ln \left(1 + q_{crit} k \frac{1+BC}{BV_{pot}} \right) + \right. \\ \left. + 1/B - V_{pot}/k \left[q_{crit} (1+BC) - \frac{BV_{pot}}{k} \right] \right\}. \quad (25)$$

From the calculating equations obtained we constructed curves describing the neutralization of a charged particle in apparatus containing disperse systems under different modes of operation of the technological installation and the neutralizer (Fig. 2).

Let us consider the motion of ions under the influence of the electric field of a charged particle. For simplicity we assume that the electric field and the diffusion act along the same straight line. If the ions are distributed along the x direction with a gradient dn/dx , then their average diffusional velocity is

$$V_{av,dif} = - \frac{D}{n} \frac{dn}{dx}. \quad (26)$$

The average velocity of ions under the influence of the electric field is $V_{av,el} = kE$, and then their velocity under the influence of both forces is

$$V_{av} = V_{av,el} + V_{av,dif} = kE - \frac{D}{n} \frac{dn}{dx}. \quad (27)$$

For $E = \text{const}$ and $V_{av} = 0$ the solution of this equation has the form

$$n = n_0 \exp \frac{kE}{D} x = n_0 \exp \frac{eE}{k_0 T} x = n_0 \exp eV/k_0 T. \quad (28)$$

Equation (28) was obtained with allowance for the fact that at small values of the field strength an ion is in thermal equilibrium with the air:

$$k/D = e/k_0T. \quad (29)$$

Equation (29) shows that the ion concentration at a point with a potential V is uniquely determined by the ratio of the electrostatic (potential) energy to the thermal (kinetic) energy.

The assumption that $E = \text{const}$ is fully justified, since we are interested in the variation of the ion concentration at a small distance from a particle, where the value of E can be taken as equal to

$$E = q/4\pi\epsilon_0r_p^2. \quad (30)$$

As a consequence of thermal motion, a charge

$$q = \beta\rho u/4. \quad (31)$$

strikes a unit of particle surface per unit time. We assume that all collisions lead to charge transfer, i.e., $\beta = 1$. The value of u is determined from the equation

$$u = \sqrt{8k_0T/\pi m}. \quad (32)$$

Thus, the change in the charge of a particle due to the thermal motion of ions will be

$$\partial^2q/\partial S\partial t = -\frac{1}{4} u n e. \quad (33)$$

Since $dS = 2\pi r_p^2 \sin\varphi d\varphi$,

$$\partial^2q/\partial\varphi\partial t = -0.5\pi u e r_p^2 n_0 \exp(eq/4\pi\epsilon_0k_0r_pT) \sin\varphi. \quad (34)$$

Let us solve this equation for φ . For weakly charged particles ($q_{\text{init}} \leq q_{\text{crit}}$), considering that φ varies from 0 to $\arccos(-kq/4\pi\epsilon_0V_{\text{pot}}R^2)$, we obtain

$$dq/dt = -A_1 \exp B_1 q (q/q_{\text{crit}} + 1), \quad (35)$$

where $A_1 = 0.5\pi u e r_p^2 n_0$, $B_1 = e/4\pi\epsilon_0k_0r_pT$, and $q_{\text{crit}} = 4\pi\epsilon_0V_{\text{pot}}R^2/k$. Taking $q = \alpha_1 q_{\text{crit}}$, where $0 \leq \alpha_1 \leq 1$, we find

$$q(t) = -\frac{1}{B_1} \ln [(\alpha_1 + 1) A_1 B_1 t + \exp(-B_1 q_{\text{init}})]. \quad (36)$$

And for strongly charged particles ($q \geq q_{\text{crit}}$) the angle is $0 \leq \varphi \leq \pi$:

$$dq/dt = -A_1 \exp B_1 q. \quad (37)$$

It is seen from Fig. 3 that the neutralization of small charged particles takes place far slower than that of large ones. This is explained by the difference in the value of $\text{grad } \rho$ and thereby of the diffusional current at the surfaces of small and large charged particles. Such a difference can be explained by assuming that in this case the diffusional current is directed opposite to the neutralization current. Finally,

$$q(t) = -\frac{1}{B_1} \ln [2A_1 B_1 t + \exp(-B_1 q_{\text{init}})]. \quad (38)$$

The value of t_{crit} , when the charge decreases from q_{init} to q_{crit} , equals

$$t_{\text{crit}} = \frac{1}{2A_1 B_1} [\exp(-B_1 q_{\text{crit}}) - \exp(B_1 q_{\text{init}})]. \quad (39)$$

The time in which the charge falls from q_{crit} to 0 is

$$t_0 = 1 - \exp(-B_1 q_{\text{crit}}) / (\alpha_1 + 1) A_1 B_1. \quad (40)$$

The total neutralization time is

$$t_{\text{tot}} = t_{\text{crit}} + t_0. \quad (41)$$

Curves of the neutralization of the charges of particles of different sizes at different densities of ions surrounding the particle in accordance with the two mechanisms of neutralization of charged particles discussed are presented in Figs. 2 and 3. As seen from the figures, the time of neutralization of charged particles due to the directional motion of ions depends on the density of ions surrounding the particle and does not depend on the particle diameter. And this time proves to be far longer than the time of neutralization due to the thermal motion and electrostatic diffusion of ions (Fig. 3).

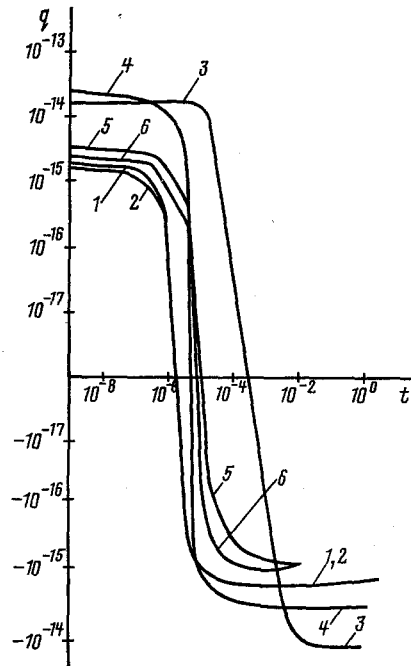


Fig. 3. Variation of charge q , C, of a particle with time t , sec (second neutralization mechanism):

Curve number	q_{crit} , C	d_p , m	n_e , m^{-3}	V_{pot} , m/sec	α
1	$0,347 \cdot 10^{-14}$	$0,49 \cdot 10^{-4}$	$0,1 \cdot 10^{-4}$	5	$0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$
2	$0,104 \cdot 10^{-14}$	$0,49 \cdot 10^{-3}$	$0,1 \cdot 10^{-4}$	1,5	$0,99 \cdot 10^{-3}$ $0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$
3	$0,347 \cdot 10^{-12}$ $0,104 \cdot 10^{-12}$	$0,49 \cdot 10^{-3}$	$0,1 \cdot 10^{-4}$	5 1,5	$0,99 \cdot 10^{-3}$ $0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$
4	$0,347 \cdot 10^{-12}$ $0,1 \cdot 10^{-12}$	$0,49 \cdot 10^{-3}$	$0,99 \cdot 10^{-12}$	5 1,5	$0,99 \cdot 10^{-3}$ $0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$
5	$0,347 \cdot 10^{-14}$	$0,49 \cdot 10^{-4}$	$0,99 \cdot 10^{-12}$	5	$0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$ $0,99 \cdot 10^{-3}$
6	$104 \cdot 10^{-14}$	$0,49 \cdot 10^{-4}$	$0,99 \cdot 10^{-12}$	1,5	$0,99 \cdot 10^{-1}$ $0,99 \cdot 10^{-2}$ $0,99 \cdot 10^{-3}$

NOTATION

e , charge of a singly charged ion; \bar{j} , vector of ion flux density; \bar{dS} , unit vector of particle surface; E_{Σ} , total field strength at particle surface; n , ion concentration near particle surface; D , coefficient of diffusion of ions; k , mobility of ions; E_e , external field strength; E_{pol} , polarization field strength of particles; E_c , strength due to Coulomb attractive force; E_m , mirror-image field strength; E_{equ} , conditional field strength equivalent to the directional force acting on an ion due to the flow of a stream of ionized air over the particle; q , charge of particle; N , particle concentration; r_i , coordinate of point where neutralization is being considered; θ , angle between external field strength vector and axis of directional ion flux to particle surface; ϵ , ϵ_p , relative permittivities of particle material and air; r_p , particle radius; R , distance between an ion and center of a particle; V_{pot} , velocity of ion motion; φ_a , angle outside which E_{Σ} is equal to zero; ρ , ion density; r_{app} , distance from center of apparatus to location of particle; n_0 , ion concentration at a sufficiently large distance from a charged particle; k_0 , Boltzmann constant; T , absolute temperature; V , field potential of a charged particle; u , average ion velocity; β , ratio of number of ions held on the surface to total number of ions colliding with a particle; m , ion mass; t_{crit} , time in which charge decreases from q_{init} to q_{crit} ; t_0 , time in which charge decreases from q_{crit} to 0; t_{tot} , total time of neutralization.

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NON-STEADY-STATE SORPTION PROCESSES IN MICROHETEROGENEOUS DISPERSED SYSTEMS*

A. Vain

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The article formulates and solves the problem of asymptotic rapid mass transfer in microheterogeneous dispersed systems.

It is accepted practice to characterize steady-state transfer of a dissolved active component (AC) through a motionless layer of a liquid or solid dispersion by the diffusion coefficient D_* . This is formally determined as the material coefficient in Fick's first law $J = -D_* \nabla c_D$, where c_D is the local concentration of the AC in the dispersed system. However, experiments with some dispersed systems type molecular sieve [1, 2] or hard polymers [3] show that Fick's second law $\nabla \cdot (D_* \nabla c_D) = D_t c_D$ does not always provide an adequate description of the non-steady-state sorption processes in dispersed systems. An analogous conclusion was reached in the study of non-steady-state heat exchange in dispersed systems [4]. The deviations are of relaxational nature. We will henceforth characterize such processes by the material parameters of the dispersed system, viz., the relaxation time λ_S .

Deviations from the predictions of the classical theory and from the actual occurrence of sorption processes may be expected in dispersed systems for which the diffusion coefficient of the dispersed phase is lower than the diffusion coefficient of the continuous phase, but the sorptivity is concentrated in the dispersed phase. The deviations are particularly great upon sudden or very rapid (compared with λ_S) changes in the concentrations of the diffusing component in the dispersed system. Relaxation phenomena, noted in molecular sieves and other solid substances with polydispersed internal structure [1], may manifest themselves in liquid dispersed systems with analogous properties during rapidly occurring processes of mass transfer, e.g., in electrochemical measurements of the diffusion coefficients [5] or in their determination by methods of a free jet, wetted wall [6], or in industrial processes of contacting gases with suspensions.

The present work presents the asymptotic description of very rapid concentration processes in dispersed systems, i.e., it examines a special, asymptotic case of more general theories [1, 2]. It was found that such asymptotic models can be formulated independently of the internal geometric structure of the medium which in the models is represented only by the specific volume of the dispersed phase φ and its specific surface ψ . In view of the variety of physical situations, the transport model for different types of dispersed systems is presented in a more general form on the basis of the concepts of microheterogeneous dispersed systems.

Microheterogeneous Dispersed Systems. They have an internal structure whose microscopic scale is fairly large compared with the molecular dimensions. Yet it is still small compared with the typical macroscopic dimensions of test specimens [7]. Separate microscopically distinguishable details of dispersed systems may be viewed as homogeneous volumetric phases of a heterogeneous polyphase system. It is expedient to study the macroscopic behavior of dispersed systems as one entity by methods of the physics of the continuum. A similar dualism of the physics of the microheterogeneous continuum manifests itself particularly

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